

Properties of Scalar & Vector

Potential :-

These are following properties of scalar and vector potential :-

1. The vector & scalar potential always satisfy the condition of vector calculus i.e. the divergence and curl of a vector quantity is always equal to zero and the grad. and Div. of a scalar quantity is always equal to zero.

2. They give non-unique value by the external electric and magnetic field by at some particular value of electric and magnetic field they give unique value.

3. They always satisfy the condition of Gauge, Lorenz and Coulomb Gauge always based upon the scalar and vector potential.

Concept of Gauge :-

From the condition of Gauge, if the gradient of a quantity is subtracted from the vector potential then there is no change in the electric field of scalar and vector potentials.

Let Ω is a quantity whose gradient is subtracting from the vector potential and in such a condition the vector potential is A' then

$$A' = A - \text{grad} \cdot \Omega \quad \left\{ \begin{array}{l} \text{This is the cond}^n \\ \text{of invariant for} \\ \text{vector potential } A \end{array} \right.$$

Now from the condition of vector potential we know that

$$B = \text{curl} A$$

For a new (vector) potential the eqⁿ can be written as

$$B' = \text{curl} A'$$

Then

$$B' = \text{curl} (A - \text{grad} \Omega)$$

$$= \text{curl } A - 0$$

$$\boxed{B' = \text{curl } A = B} \quad \text{--- (1)}$$

This means that there is no change in the magnetic field.

Now we know that

$$E = -\text{grad } \phi - \frac{\partial A}{\partial t}$$

For new vector (A') and scalar potential (ϕ'), then above eqⁿ can be written as

$$E' = -\text{grad } \phi' - \frac{\partial A'}{\partial t}$$

$$= -\text{grad } \phi' - \frac{\partial}{\partial t} (+A - \text{grad } \Omega)$$

$$= -\text{grad} \left(\phi' - \frac{\partial \Omega}{\partial t} \right) - \frac{\partial A}{\partial t}$$

Now let us define

$$\boxed{\phi' = \phi + \frac{\partial \Omega}{\partial t}}$$

This is the one of invariance for scalar potential

$$E' = -\text{grad} \left(\phi + \frac{\partial \Omega}{\partial t} - \frac{\partial \Omega}{\partial t} \right) - \frac{\partial A}{\partial t}$$

$$\vec{E}' = -\text{grad } \phi - \frac{\partial A}{\partial t} = \vec{E}$$

From eqⁿ ① and ② it is clear that if the vector and scalar potential is changed then there is no change in the electric field and magnetic field.